# MODEL PAPER

## MATHEMATICS GRADE-12 (2nd Year)

BISE Examination 2027 — Smart Syllabus (ALP) — Paper Code: Math-XII

**Time Allowed:** 3 Hours **Total Marks:** 100

**Instructions:**

(1) Q. 1 (MCQs) is compulsory; each MCQ carries one mark. Cutting or overwriting an answer cancels it.

(2) Part-I: Attempt any **8** of 12 parts in Q.2, any **8** of 12 parts in Q.3 and any **9** of 13 parts in Q. 4. Each part carries 2 marks.

(3) Part-II: Attempt any **THREE** questions from Q. 5 to Q. 9. Each question has two parts i.e. (a) and (b) that carries 5 marks each.

(4) A non-programmable scientific calculator may be used. Take $g = 9.8$ m/s$^2$ wherever needed.

### OBJECTIVE (MCQs) — Marks: 20

**Q. 1 Choose the correct option. Each MCQ carries one mark.** **(1 × 20 = 20)**

| No. | Question | A | B | C | D |
|---|---|---|---|---|---|
| 1 | The $y$-intercept of the graph of $y = 2(x+1)(x-3)$ is: | $6$ | $-3$ | $-6$ | $3$ |
| 2 | If the graph of $y = f(x)$ is shifted 3 units upward, the new function is: | $f(x+3)$ | $f(x-3)$ | $3f(x)$ | $f(x)+3$ |
| 3 | If $y = \tan^{-1}(2x)$, then $\frac{dy}{dx} =$ | $\frac{2}{1+4x^2}$ | $\frac{1}{1+4x^2}$ | $\frac{2}{1+2x^2}$ | $\frac{2x}{1+4x^2}$ |
| 4 | If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx} =$ | $t$ | $2t$ | $\frac{1}{t}$ | $\frac{2}{t}$ |
| 5 | The function $f(x) = x^3 - 3x$ has a relative maximum at $x =$ | $1$ | $-1$ | $0$ | $3$ |
| 6 | $\int \cos 3x\, dx =$ | $3\sin 3x + c$ | $-1/3 \sin 3x + c$ | $-3\sin 3x + c$ | $1/3 \sin 3x + c$ |
| 7 | $\int \frac{f'(x)}{f(x)}\, dx =$ | $\ln\lvert f(x)\rvert + c$ | $f(x) + c$ | $\frac{[f(x)]^2}{2} + c$ | $\frac{1}{f(x)} + c$ |
| 8 | The area bounded by $y = x^2$, the $x$-axis and the lines $x = 0$, $x = 3$ is: | 3 | 6 | 9 | 27 |
| 9 | The differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is: | separable | homogeneous | of order 2 | of degree 2 |
| 10 | The slope of the line $3x - 4y + 7 = 0$ is: | $\frac{3}{4}$ | $-\frac{3}{4}$ | $\frac{4}{3}$ | $-\frac{4}{3}$ |

| No. | Question | A | B | C | D |
|---|---|---|---|---|---|
| 11 | The lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular if: | $a - b = 0$ | $h^2 = ab$ | $h = 0$ | $a + b = 0$ |
| 12 | The centre of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ is: | $(-3,2)$ | $(3,-2)$ | $(6,-4)$ | $(-6,4)$ |
| 13 | The eccentricity of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is: | $\frac{4}{3}$ | $\frac{3}{5}$ | $\frac{5}{3}$ | $\frac{5}{4}$ |
| 14 | A car starts from rest with uniform acceleration 2 m/s$^2$. Its velocity after 5 s is: | 2.5 m/s | 7 m/s | 25 m/s | 10 m/s |
| 15 | If the position of a particle is $x(t) = t^3 - 2t$, its velocity at $t = 2$ s is: | 10 | 4 | 12 | 6 |
| 16 | By the Location of Root Theorem, a root of $x^3 - x - 4 = 0$ lies in: | $(0,1)$ | $(1,2)$ | $(2,3)$ | $(3,4)$ |
| 17 | The principal value of $\sin^{-1}(-1/2)$ is: | $\frac{5\pi}{6}$ | $\frac{7\pi}{6}$ | $-\frac{\pi}{6}$ | $\frac{11\pi}{6}$ |
| 18 | The solutions of $\tan x = 1$ in $[0,2\pi]$ are: | $\frac{\pi}{4}$ only | $\frac{\pi}{4}, \frac{3\pi}{4}$ | $\frac{3\pi}{4}, \frac{7\pi}{4}$ | $\frac{\pi}{4}, \frac{5\pi}{4}$ |
| 19 | If $\cos x = -1/2$ and $x$ lies in the third quadrant, then $x =$ | $\frac{4\pi}{3}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{3}$ | $\frac{7\pi}{6}$ |
| 20 | The domain of $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \ln(t-1)\,\mathbf{j}$ is: | $[0,\infty)$ | $(1,\infty)$ | $(0,\infty)$ | $[1,\infty)$ |

**SUBJECTIVE — PART-I (Short Questions) — Marks: 50**

**Q. 2 Attempt any EIGHT parts.** **(2 × 8 = 16)**

(i) Prove the following hyperbolic identity: $1 - \tanh^2 x = \operatorname{sech}^2 x$.

(ii) Given the function $f(x) = \sqrt{x}$, find the equation of the function after vertical stretch by factor 2.

(iii) A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 30 km/h in 5 seconds. Find the distance travelled by the particle in the last three seconds.

(iv) The acceleration of a particle is $a(t) = \frac{1}{1+t}$ for $t \geq 0$. If the velocity is 0 m/s when $t = 0$, find the velocity at $t = 5$ seconds.

(v) Differentiate $y = x^2 \cos x$ with respect to the variable involved.

(vi) Find $f'(x)$ if $f(x) = \sin(2^x)$.

(vii) Find $y_2$ if $y = x^4 - 3x^2 + 4x - 5$.

(viii) Find the linear approximation of the function $f(x) = \sqrt{x^2 + 16}$ at $x = 3$.

(ix) Find the differential $dy$ if $y = x^3 - 3\sqrt{x}$.

(x) Show that the relation $y = cx - 1$ is a solution of the differential equation $x\frac{dy}{dx} = 1 + y$.

(xi) Find the general solution of the differential equation $y\,dx + x\,dy = 0$.

(xii) A particle is moving in a straight line and its acceleration is given by $a = 2t - 7$. Find velocity $v$ in terms of $t$ if $v = 10$ m/sec, when $t = 0$.

**Q. 3 Attempt any EIGHT parts.** **(2 × 8 = 16)**

(i) Find the integral, $\int x^2 \left(x + \frac{1}{x}\right) dx$.

(ii) Find the anti-derivative $\int \frac{e^x}{1+e^{2x}}\, dx$.

(iii) Evaluate $\int \ln x\, dx$.

(iv) Solve by partial fraction $\int \frac{5x+7}{(x+1)(x+2)}\, dx$.

(v) Evaluate the definite integral $\int_1^2 (x^2 + x)\, dx$.

(vi) For what value of $k$ if the lines $x + y - 4 = 0$, $2x - y + 1 = 0$ and $kx + 2y - 10 = 0$ are concurrent.

(vii) Find the area of the triangle formed by the points $A(-3,2)$, $B(4,-1)$, $C(1,5)$.

(viii) Workout the value of $k$ such that the lines represented by the equation $(k-8)x^2 - 9xy + 3ky^2 = 0$ are perpendicular to each other.

(ix) Find an equation of the circle with centre at $(5,7)$ and radius 2.

(x) Find the equations of tangents to the circle $x^2 + y^2 = 4$ which are parallel to the line $3x + 4y + 1 = 0$.

(xi) Write the equation of the parabola with focus $(-3,2)$ and directrix $x = 5$.

(xii) Find the length of the latus rectum of the hyperbola $9x^2 - 16y^2 = 144$.

**Q. 4 Attempt any NINE parts.** **(2 × 9 = 18)**

(i) Find the real root of $f(x) = x^3 - x - 1$ correct to two decimal places using the graphical method, by finding some interval of sign change.

(ii) Find the real root of $f(x) = x^3 - 3x - 10$ correct to 3 decimal places using the bisection method, within the interval of sign change [2,3].

(iii) Use Trapezoidal Rule to approximate the value of $\int_0^4 \sqrt{x}\ dx$ taking $n = 4$.

(iv) Find the principal value of $\tan^{-1}(-\sqrt{3})$.

(v) Find the domain of $f(x) = \sin^{-1}(5x - 3)$.

(vi) Prove that $\cos(\sin^{-1}x) = \sqrt{1 - x^2}$.

(vii) Solve the trigonometric equation $\sin x = \frac{1}{\sqrt{2}}$.

(viii) Solve the trigonometric equation $\sqrt{2}\cos x \sin x - \sin x = 0$.

(ix) A person standing 20 meters away from the base of a tree measures the angle of elevation to the top of the tree as 30°. Find the height of the tree.

(x) Give two real-life examples where vector-valued functions are necessary.

(xi) Given $x(t) = t^2 + 1$, $y(t) = \sqrt{t}$, construct $\mathbf{r}(t)$.

(xii) Find the first-order derivative of $\mathbf{r}(t) = (t^2 + 1)\hat{\imath} + (t^3 + t^2 + 3t)\hat{\jmath}$ with respect to $t$.

(xiii) Find the first-order derivative of $\mathbf{r}(t) = te^{-t}\,\hat{\imath} + \cos(3t)\,\hat{\jmath} + \ln(1 + t^2)\hat{k}$ with respect to $t$ at $t = 0$.

**SUBJECTIVE — PART-II (Long Questions) — Marks: 30**

**Attempt any THREE questions. Each part carries 5 marks.** **[3 × (5 + 5) = 30]**

**Q. 5** (a) Solve the equation: $3^{(x+1)} - 2 \cdot 9^x + 9 = 0$.

(b) Prove that: $\sin^{-1}\frac{12}{13} - \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{33}{65}$.

**Q. 6** (a) Using the Second Derivative Test, find local extreme values of $f(x) = x^4 - 8x^2 + 2$.

(b) Solve the initial value problem: $\frac{dy}{dx} - x = xy^2;\ y(0) = 1$.

**Q. 7** (a) Find two positive integers whose sum is 12 and the product of one with the square of the other will be maximum.

(b) The vertices of $\triangle PQR$ are $P(1,2)$, $Q(5,-6)$ and $R(7,-2)$. Find the equations of all three medians.

**Q. 8** (a) Find the area bounded by the parabola $y = x^2 - 6x + 8$, $x = 2$, $x = 4$ and the $x$-axis.

(b) A particle is projected vertically upwards with a velocity 100 m/s, and another is let fall from a height of 200 m at the same time. Find the height of the point where they meet each other.

**Q. 9** (a) Find the equation of the ellipse as the locus of points $P(x, y)$ such that the sum of the distances from $P$ to the points $(-3,0)$ and $(3,0)$ is 10.

(b) Find the real root of the nonlinear equation $e^x = 4x$ near $x = 2$, accurate to 3 decimal places, using the Newton–Raphson method.